Bayesian Statistics
(a very brief introduction)

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Epi 516, Biost 520

1.30pm, T478, April 4, 2018

NOTE: This document has been modified.

Overview

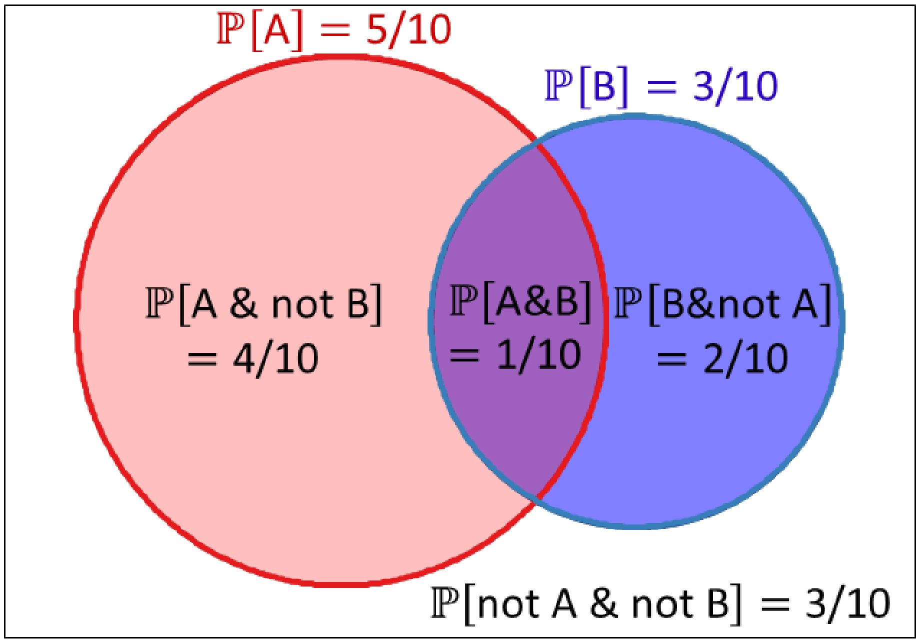
Using a spade for some jobs and a shovel for others does not require you to sign up to a lifetime of using only Spadian (Bayesian) or Shovelist (Null Hypothesis Significance Testing) philosophy or to believing that only spades or only shovels represent the One True Path to garden neatness.

There are different ways of tackling statistical problems, too.

Bayes' Theorem

Before we get to inference: Bayes' *Theorem* is a result in conditional probability, stating that for two events A and B...

$$\mathbb{P}[\,A|B\,] = \frac{\mathbb{P}[\,A \text{ and } B\,]}{\mathbb{P}[\,B\,]} = \mathbb{P}[\,B|A\,]\frac{\mathbb{P}[\,A\,]}{\mathbb{P}[\,B\,]}.$$



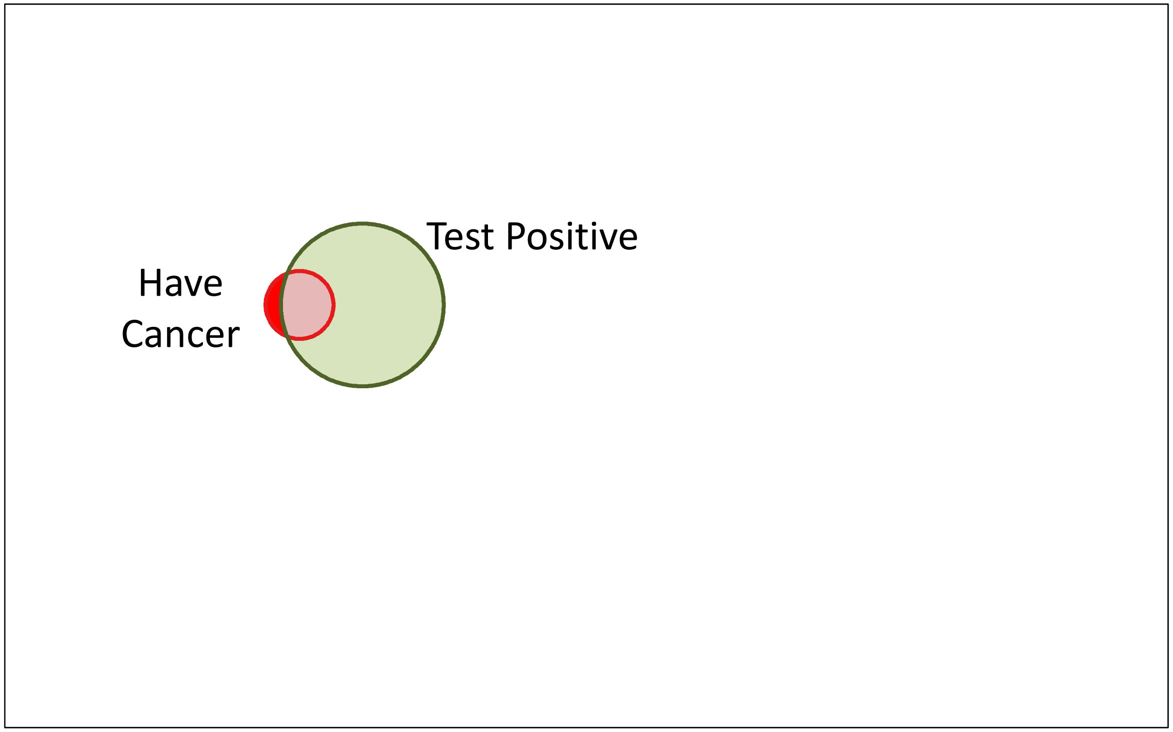
In this example;

- $\mathbb{P}[\,A|B\,] = \frac{1/10}{3/10} = 1/3$
- $\mathbb{P}[\,B|A\,] = \frac{1/10}{5/10} = 1/5$
- And $1/3 = 1/5 \times \frac{5/10}{3/10}$ (✓)

In words: the conditional probability of A given B is the conditional probability of B given A scaled by the *relative* probability of A compared to B.

Bayes' Theorem

If 1% of a population have cancer, for a screening test with 80% sensitivity, which means that 80% of the time it will correctly detect patient who have cancer, and the same screening test has 95% specificity, which means that 95% of the time it will correctly detect patients who do not have cancer:



$$\begin{aligned} \mathbb{P}[\,\text{Test } +\text{ve} | \text{Cancer}\,] &= 80\% \\ \frac{\mathbb{P}[\,\text{Test } +\text{ve}]}{\mathbb{P}[\,\text{Cancer}\,]} &= 5.75 \\ \mathbb{P}[\,\text{Cancer} | \text{Test } +\text{ve}\,] &\approx 14\% \end{aligned}$$

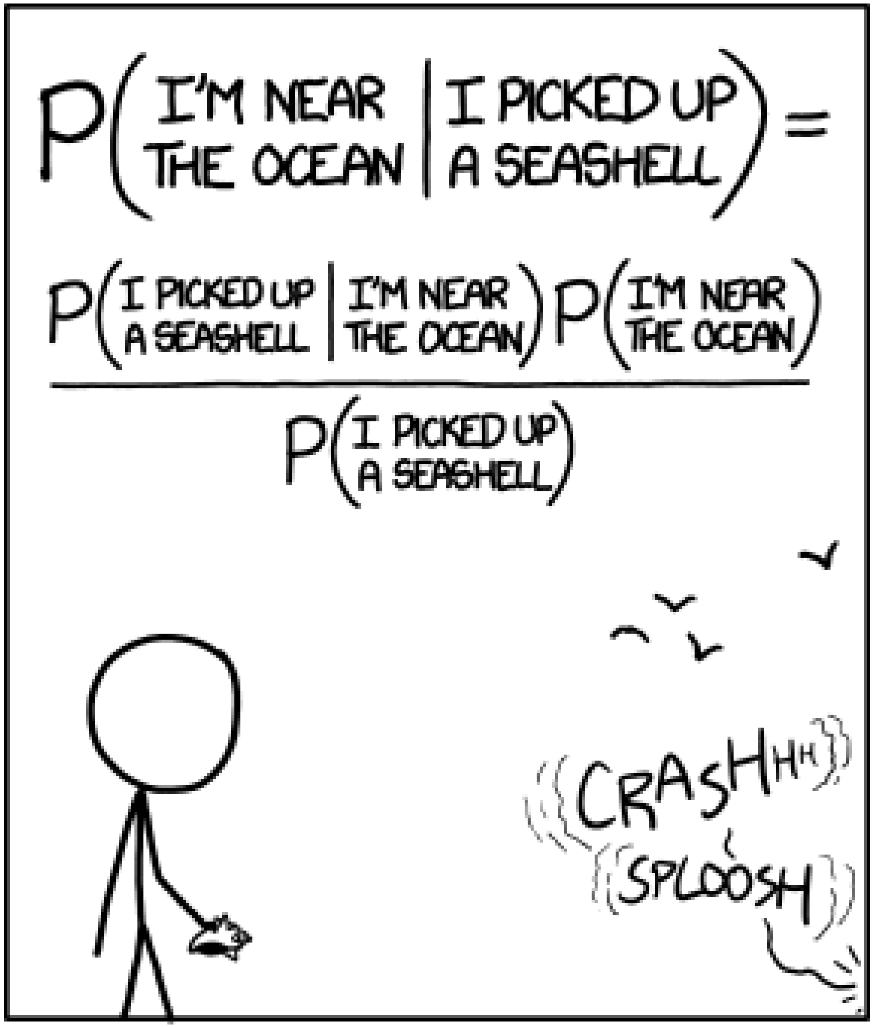
... i.e. most positive results are actually false alarms

Mixing up $\mathbb{P}[\,A|B\,]$ with $\mathbb{P}[\,B|A\,]$ is the Prosecutor's Fallacy; a small probability calculated for evidence (*A*) assuming the suspect is innocent (*B*) need NOT mean a small probability calculated for the suspect being innocent (*B*) given the probability calculated for the evidence (*A*).

Bayes' Theorem: Sally Clark

- After the sudden death of two baby sons, Sally Clark (pictured above) was sentenced to life in prison in 1999.
- Among other errors, expert witness Prof Roy Meadow (pictured above) had wrongly interpreted the small probability of two crib deaths occurring (small probability calculated for evidence) as a small probability of Clark's innocence.
  - After a long campaign, including refutation of Meadow's statistics, Clark was released and cleared in 2003.
  - After being freed, she developed alcoholism and died in 2007.

Bayes' Theorem: XKCD at the beach



STATISTICALLY SPEAKING, IF YOU PICK UP A SEASHELL AND *DON'T* HOLD IT TO YOUR EAR, YOU CAN PROBABLY HEAR THE OCEAN.

This is roughly equal to

$$\frac{\textit{\# of times I've picked up a seashell when I'm near the ocean}}{\textit{\# of times I've picked up a seashell}}$$

Bayesian inference

So far, nothing's controversial; Bayes' Theorem is a rule about the 'language' of probabilities, that can be used in any analysis describing random variables, i.e. any data analysis.

Q. So why all the fuss?
A. Bayesian *inference* uses more than just Bayes' Theorem

> In *addition* to describing random variables, Bayesian inference uses the 'language' of probability to describe what is known about parameters.

Note: Null hypothesis significance testing, using p-values & confidence intervals, does not quantify what is known about parameters.*

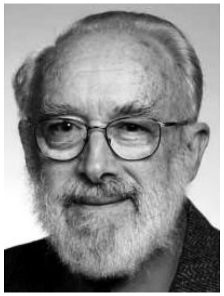
*many people initially think it does, which is an important job for instructors of intro Stat/Biostat courses is point this out.

Fight! Fight! Fight!

Two old-timers slugging out the Bayes *vs* NHST battle;

"If [Bayesians] would only do as [Bayes] did and publish posthumously we should all be saved a lot of trouble"

Maurice Kendall (1907–1983), JRSSA 1968



"The only good statistics is Bayesian Statistics"

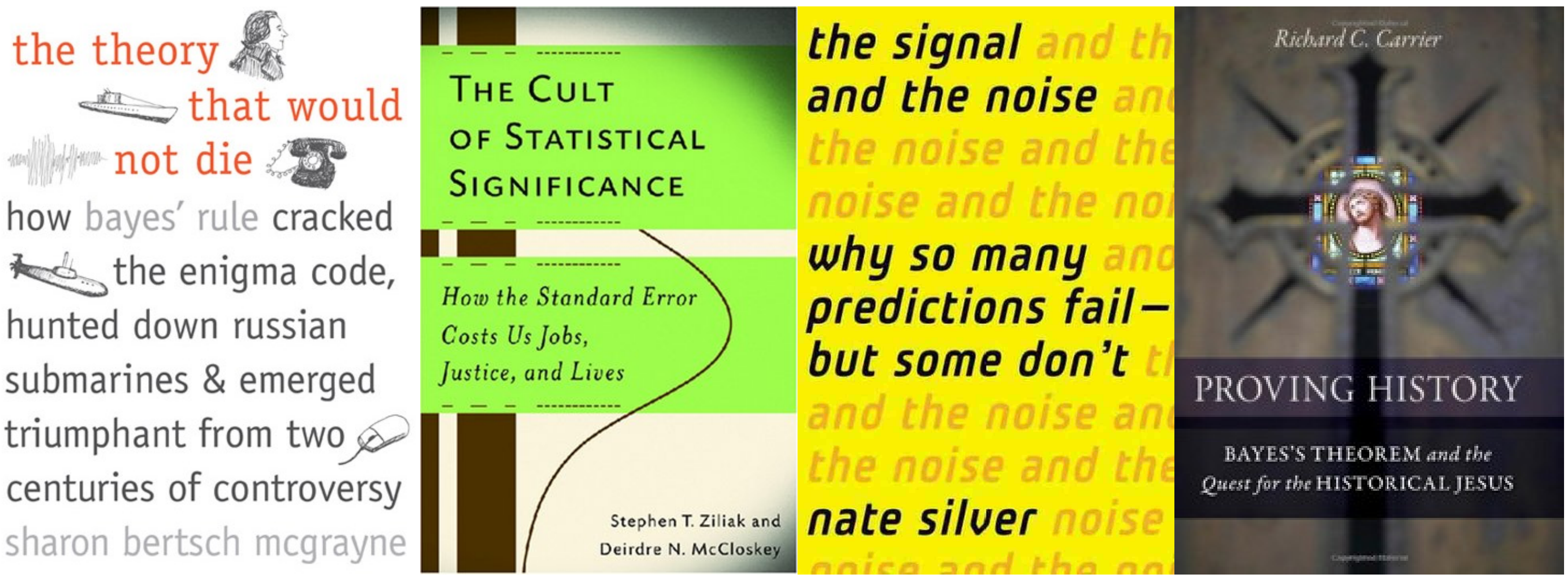
Dennis Lindley (1923–2013)
in The Future of Statistics: A Bayesian 21st Century (1975)

- For many years – until recently – Bayesian ideas in statistics* were widely dismissed, often without much thought.
- Advocates of Bayes had to fight hard to be heard, leading to an 'us against the world' mentality – & predictable backlash.
- Today, debates *tend* be less acrimonious, and more tolerant.

* *and sometimes the statisticians who researched and used them*

Fight! Fight! Fight!

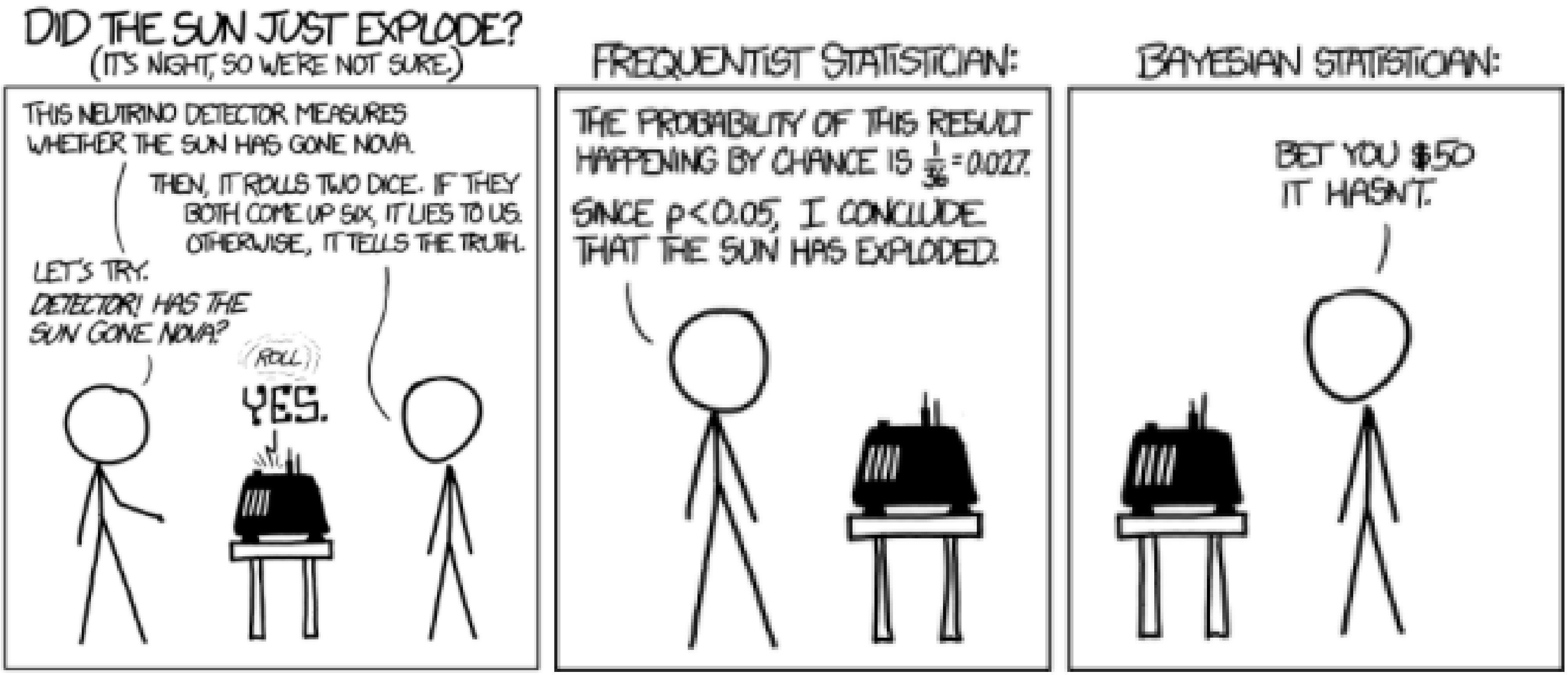
But writers of dramatic/romantic stories about Bayesian "heresy" tend (I think) to over-exaggerate the actual differences (as shown in these book covers)



- Among those who actually understand both, it's hard to find people who totally dismiss either one.
- Keen people: Vic Barnett's Comparative Statistical Inference provides the most even-handed exposition I know.

Fight! Fight! Fight!

The science cartoon XKCD again, on Frequentists statisticians who practice Null Hypothesis Significance Testing) vs Bayesians:



Here, the fun relies on setting up a straw-man; p-values are not the only tools used in a *skillful* null hypothesis significance testing.

Note: As you know, statistics can be *hard* – so it's not difficult to find examples where it's done badly, under any system.

Summary

Bayesian statistics:

- Is useful in many settings, and you should know about it.
- Is often not very different in practice from null hypothesis significance testing statistics; it is often helpful to think about analyses from both Bayesian and non-Bayesian points of view.
- Is not reserved for hard-core mathematicians, computer scientists, or philosophers. If you find it helpful, use it.